



Advanced Topics in Grid & Gas Modeling and Reliability

Misha Chertkov

LANL/DOE:OE + LANL/DTRA & NMC/NSF:ECCS

Los Alamos National Laboratory & New Mexico Consortium

Jan 12, 2015 LANL Grid Science School

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

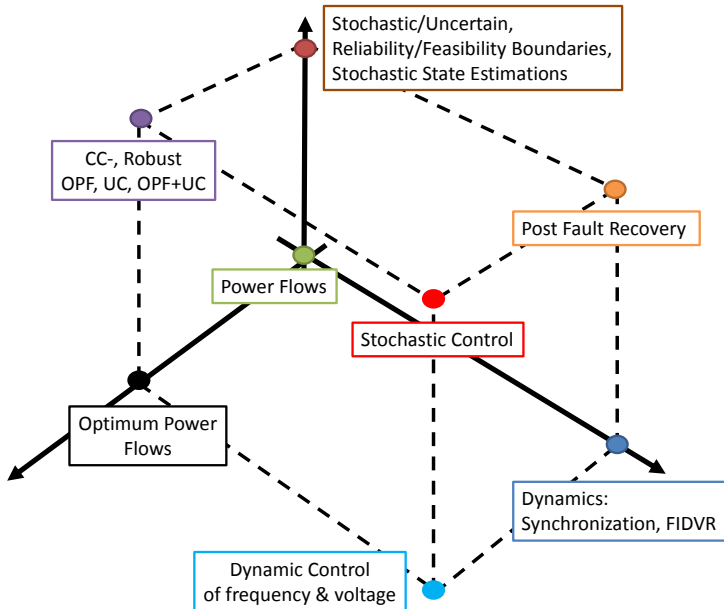
- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges



Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

The Kirchhoff Laws (linear)

$$\forall i \in \mathcal{G}_0 : \sum_{j \sim i} I_{ij} = I_i \text{ for currents}$$

$$\forall (i, j) \in \mathcal{G}_1 : I_{ij} z_{ij} = V_i - V_j \text{ for voltages}$$

$$\Rightarrow \forall (i, j) \in \mathcal{G}_1 : I_i = \sum_{j \in \mathcal{G}_0} Y_{ij} V_j$$

$$\hat{Y} = (Y_{ij} | i, j \in \mathcal{G}_0), \quad \forall \{i, j\} : Y_{ij} = \begin{cases} 0, & i \neq j, \quad i \not\sim j \\ -y_{ij}, & i \neq j, \quad i \sim j \\ \sum_{k \neq i}^{k \sim i} y_{ik}, & i = j. \end{cases}$$

$$\forall \{i, j\} : y_{ij} = g_{ij} + \hat{i} \beta_{ij} = (z_{ij})^{-1}, \quad z_{ij} = r_{ij} + x_{ij}$$

Complex Power Flows [balance of power, nonlinear]

$$\begin{aligned} \forall i \in \mathcal{G}_0 : \quad p_i + \hat{i} Q_i &= V_i I_i^* = V_i \sum_{j \sim i} I_{ij}^* = V_i \sum_{j \sim i} \frac{V_i^* - V_j^*}{z_{ij}^*} \\ &= \sum_{j \sim i} \frac{\exp(2\rho_i) - \exp(\rho_i + \rho_j + \hat{i}\theta_i - \hat{i}\theta_j)}{z_{ij}^*} = V_i \sum_j Y_{ij}^* V_j^* \end{aligned}$$

- Flows on graphs, but very different from transportation networks
- Nonlinear in terms of Real and Reactive powers
- Known parameters: different (injection/consumption/control) conditions on generators (p, v) and loads (p, q) . The task is to find the unknown (flows and voltages).
- Simplified a bit (transformers, shunts, etc, can also be accounted for)

$$V = v \exp(i\theta), \quad \underbrace{z}_{\text{impedance}} = \underbrace{r}_{\text{resistance}} + \hat{i} \underbrace{x}_{\text{inductance}}, \quad \underbrace{z^{-1}}_{\text{admittance}} = \underbrace{g}_{\text{conductance}} + \hat{i} \underbrace{\beta}_{\text{susceptance}}$$

- └ Power Flows (advanced topics/methods/techniques)

- └ Energy Function

Outline

Power Flows (advanced topics/methods/techniques)

Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

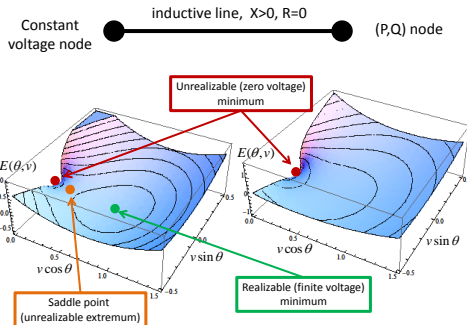
└ Power Flows (advanced topics/methods/techniques)

└ Energy Function

Lossless ($r/x \rightarrow 0$)

Power Flow Eqs. \leftarrow variation of Energy Function

$$E(\theta, \mathbf{v}) = - \sum_i p_i \theta_i - \sum_{i \in \text{loads}} q_i \log v_i + \sum_{(i,j)} \beta_{ij} \frac{|V_i - V_j|^2}{2}$$



$\forall i :$

$$\partial_{\theta_i} E(\theta, \mathbf{v}) = 0$$

$\forall i \in \text{Loads} = (p, q) \text{ nodes} :$

$$\partial_{\rho_i} E(\theta, \mathbf{v}) = 0$$

- Stationary point(s), over **phases** and **log-voltages**, $\rho_i \doteq \log v_i$, correspond to solutions (stable or not) of the PF equations

“Nonlinear DC” = losses are ignored + voltages are fixed.

$$E(\theta) = - \sum_i P_i \theta_i + \sum_{(i,j)} \beta_{ij} (1 - \cos(\theta_i - \theta_j))$$

$$\forall i: \quad p_i = \sum_{j:(i,j)} \beta_{ij} \sin(\theta_i - \theta_j)$$

■ Solution is unique

$$\forall (i,j): \quad |\theta_i - \theta_j| < \pi/2$$

■ Energy function is convex in the domain

Dual Formulation – convex too

$$\min_{\rho \text{ - line flows}} \quad \underbrace{\sum_{(i,j)} \beta_{ij} \Psi(\rho_{ij})}_{\text{reactive losses in lines}}, \quad \Psi(\rho) = \int_{-1}^{\rho} \arcsin(y) dy$$

$$\text{s.t.} \quad \underbrace{\sum_{j:(i,j)} \beta_{ij} \rho_{ij} - \sum_{j:(j,i)} \beta_{ij} \rho_{ji}}_{\text{network flow conservation}} = P_i \quad \forall i \quad (*)$$

$$|\rho_{ij}| < 1 \quad \text{for each line } (i,j)$$

- If θ_i is the optimal dual for (*), $\rho_{ij} = \sin(\theta_i - \theta_j)$.
- Boyd & Vandenberghe (add. ex. for convex opt. – 2012)
- Corrected/clarified: R. Bent, D. Bienstock, MC - IREP 2013
- More details (and generalizations) in the conference talk by Krishnamurthy Dvijotham (Dj)

- └ Power Flows (advanced topics/methods/techniques)

- └ Distribution Flows

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

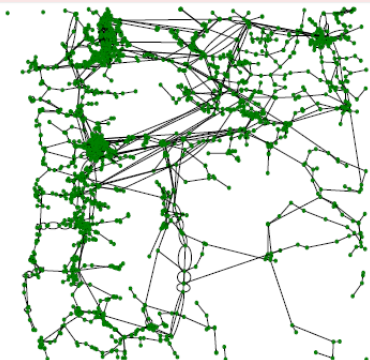
Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

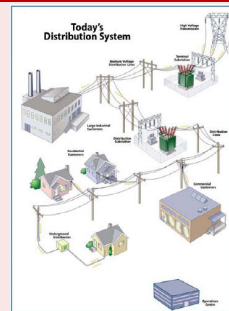
- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

Linear Structure of the Distribution



Transmission



Distribution: tree-like structure
“growing” from a transmission
node

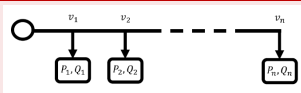
Distribution is (Especially) Prone to Nonlinear Effects

└ Power Flows (advanced topics/methods/techniques)

└ Distribution Flows

Dist(ributed) Flow Representation [Baran, Wu '89]

graph-linear Element $k = 1, \dots, N$ of the distribution feeder



$$k = 0, \dots, N, \quad v_0 = 1$$

$$\rho_{n+1} = \phi_{n+1} = 0$$

$$\rho_{k+1} - \rho_k = p_k - r_k \frac{\rho_k^2 + \phi_k^2}{v_k^2}$$

$$\phi_{k+1} - \phi_k = q_k - x_k \frac{\rho_k^2 + \phi_k^2}{v_k^2}$$

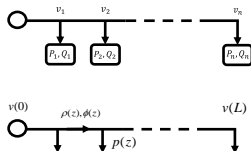
$$v_{k+1}^2 - v_k^2 = -2(r_k \rho_k + x_k \phi_k) - (r_k^2 + x_k^2) \frac{\rho_k^2 + \phi_k^2}{v_k^2}$$

- nonlinear AC over a line
- generalizable to a tree
- ρ_k, ϕ_k real and reactive powers flowing through the segment k
- p_k, q_k, v_k powers injected/consumed and voltage at the bus k

└ Power Flows (advanced topics/methods/techniques)

└ Distribution Flows

Continuum (one dimensional) static power flows



ODE with mixed boundary conditions:

$$v(0) = 1, \rho(L) = \phi(L) = 0$$

WHY ?

- Model reduction (fewer/slower parameters)
- Easier to see/analyze qualitative phenomena

From Algebraic Eqs. on a (linear) Graph to Power Flow ODEs

$$0 = \underbrace{p + \beta \partial_r \left(v^2 \partial_r \theta \right) + g v \left(\partial_r^2 v - v \left(\partial_r \theta \right)^2 \right)}_{\text{balance of real power}}, \quad 0 = \underbrace{q + \beta v \left(\partial_r^2 v - v \left(\partial_r \theta \right)^2 \right) - g \partial_r \left(v^2 \partial_r \theta \right)}_{\text{balance of reactive power}}$$

$$\rho = \underbrace{-\beta v^2 \partial_r \theta - g v \partial_r v}_{\text{real power density flowing through the segment}}, \quad \phi = \underbrace{-\beta v \partial_r v + g v^2 \partial_r \theta}_{\text{reactive power density flowing through the segment}}$$

$$0 = \underbrace{p}_{\text{real consumption}} - \underbrace{\partial_r \rho}_{\text{real transport}} - \underbrace{r \frac{\rho^2 + \phi^2}{v^2}}_{\text{real dissipation}}, \quad 0 = \underbrace{q}_{\text{reactive consumption}} - \underbrace{\partial_r \phi}_{\text{reactive transport}} - \underbrace{x \frac{\rho^2 + \phi^2}{v^2}}_{\text{reactive dissipation}}$$

- └ Power Flows (advanced topics/methods/techniques)
 - └ Linear Approximations beyond DC (coupled & decoupled)

Outline

Power Flows (advanced topics/methods/techniques)

Energy Function

Distribution Flows

Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

Direct Methods: on-line post-fault analysis

Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

Gas Dynamics – pipeline fundamentals

Static/Balanced Flows. Compression. Energy Function.

Dynamics. Line Pack. Approximations.

Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

Probabilistic State Estimations – a Gas Example

Distance to Failures. Instantons – a Grid Example.

Gas-Grid Coupling, Challenges

└ Power Flows (advanced topics/methods/techniques)

└ Linear Approximations beyond DC (coupled & decoupled)

Linear Coupled (LC) Approximation

$\forall i \in \mathcal{V} :$

$$p_i = \sum_{j:(i,j)} (\beta_{ij}(\theta_i - \theta_j) + g_{ij}(\varepsilon_i - \varepsilon_j))$$

$$q_i = \sum_{j:(i,j)} (-g_{ij}(\theta_i - \theta_j) + \beta_{ij}(\varepsilon_i - \varepsilon_j)) ,$$

$\forall i :$

$$|v_i| - 1 = \varepsilon_i \ll 1, |\theta_i| \ll 1$$

$\forall (i,j) :$

$$g_{ij} \doteq \frac{r_{ij}}{x_{ij}^2 + r_{ij}^2}, \quad \beta_{ij} \doteq \frac{x_{ij}}{x_{ij}^2 + r_{ij}^2}$$

- Linearization around $\theta = 0$ & $v = 1$ – **analytic**, **accounts for voltage**
- Linearization around a current PF solution (if known) may work better

Linear De- Coupled (DC-inductive approximation)

$\forall a \in \mathcal{V}$:

$$p_a = \sum_{b:(ab) \in \mathcal{E}^T} (\beta_{ab}(\theta_a - \theta_b) + \cancel{g_{ab}(\varepsilon_a - \varepsilon_b)})$$

$$q_a = \sum_{b:(ab) \in \mathcal{E}^T} (\cancel{-g_{ab}(\theta_a - \theta_b)} + \beta_{ab}(\varepsilon_a - \varepsilon_b))$$

- $r \ll x$ – transmission
- natural extension of the DC approximation accounting for voltage deviation from nominal
- analytic + also considered as a computational scheme
- decoupling aligns with the "jargon" separation in the two pairs, (θ, p) and (v, q)

- Power Flows (advanced topics/methods/techniques)

- Linear Approximations beyond DC (coupled & decoupled)

Dist-Flow (again) and Lin-Dist-Flow [Baran-Wu '1989]

Distributed Flow (Dist-Flow)

$$p_{a \rightarrow b} - r_{ab} \frac{p_{a \rightarrow b}^2 + q_{a \rightarrow b}^2}{v_a^2} = p_b + \sum_{(bc) \in \mathcal{E}^T; c \neq a} p_{b \rightarrow c}$$

$$q_{a \rightarrow b} - x_{ab} \frac{p_{a \rightarrow b}^2 + q_{a \rightarrow b}^2}{v_a^2} = q_b + \sum_{(bc) \in \mathcal{E}^T; c \neq a} q_{b \rightarrow c}$$

$$v_b^2 = v_a^2 - 2(r_{ab}p_{a \rightarrow b} + x_{ab}q_{a \rightarrow b}) + (r_{ab}^2 + x_{ab}^2) \frac{p_{a \rightarrow b}^2 + q_{a \rightarrow b}^2}{v_a^2}$$

- In the loopy case — # of variables is larger than # of equations \Rightarrow reproduce only a subset of PF eqs.

- Power Flows (advanced topics/methods/techniques)

- Linear Approximations beyond DC (coupled & decoupled)

Dist-Flow (again) and Lin-Dist-Flow [Baran-Wu '1989]

Linearized Distributed Flow (Lin-Dist-Flow)

$$p_{a \rightarrow b} - r_{ab} \frac{p_{a \rightarrow b}^2 + q_{a \rightarrow b}^2}{v_a^2} = p_b + \sum_{(bc) \in \mathcal{E}^T; c \neq a} p_{b \rightarrow c}$$

$$q_{a \rightarrow b} - x_{ab} \frac{p_{a \rightarrow b}^2 + q_{a \rightarrow b}^2}{v_a^2} = q_b + \sum_{(bc) \in \mathcal{E}^T; c \neq a} q_{b \rightarrow c}$$

$$v_b^2 = v_a^2 - 2(r_{ab}p_{a \rightarrow b} + x_{ab}q_{a \rightarrow b}) + (r_{ab}^2 + x_{ab}^2) \frac{p_{a \rightarrow b}^2 + q_{a \rightarrow b}^2}{v_a^2}$$

└ Power Flows (advanced topics/methods/techniques)

└ Linear Approximations beyond DC (coupled & decoupled)

Dist-Flow (again) and Lin-Dist-Flow [Baran-Wu '1989]

Linearized Distributed Flow (Lin-Dist-Flow)

$$\begin{aligned}p_{a \rightarrow b} &\approx p_b + \sum_{(bc) \in \mathcal{E}^T; c \neq a} p_{b \rightarrow c} \\q_{a \rightarrow b} &\approx q_b + \sum_{(bc) \in \mathcal{E}^T; c \neq a} q_{b \rightarrow c} \\v_b^2 &\approx v_a^2 - 2(r_{ab}p_{a \rightarrow b} + x_{ab}q_{a \rightarrow b})\end{aligned}$$

- losses of active and reactive powers are ignored & voltage degradation is small
- In distribution (tree-) **equivalent to the LC approximation**

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

- └ Power Grid Dynamics (advanced topics/methods/techniques)

- └ Direct Methods: on-line post-fault analysis

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis**

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

└ Power Grid Dynamics (advanced topics/methods/techniques)

└ Direct Methods: on-line post-fault analysis

Problem Setting

- Transmission Grid. Available list of contingencies, dependent on the current state of loads and generation.
- Majority of contingencies in the list are faults – lasting for 0.1-0.5s and then cleared.
- Direct simulations of the on-fault and post-fault dynamics for the entire list is prohibitively expensive.

Challenge & Suggested Solution

- Design a direct method — fast/efficient computations. The approach of Aylett 1958; Varaya, Wu, Chen 1985; Pai 1989 +++ with a new twist

- **convex optimization**

testing if the system survives contingency(ies)

... presentation follows the logic of arXiv:1409.4451 by S. Backhaus, R. Bent, D. Bienstock, MC, D. Krishnamurthy

- “Efficient Synchronization Stability Metrics for Fault Clearing”
- ... more like intro to the problem ... than complete solution (but getting closer :)

Basic Dynamic Equation [re-cap from Florian lecture]

[Nonlinear, Lossless]

$$M_i \ddot{\theta}_i + \gamma_i \dot{\theta}_i = p_i - \sum_{j: \{i,j\} \in \mathcal{E}} v_i v_j \beta_{ij} \sin(\theta_i - \theta_j)$$

- $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \beta)$ – vertices, edges, susceptances, lossless lines
- $p = (p_i | i \in \mathcal{V})$ – globally balanced vector of mechanical power inputs and power consumptions
- $M = (M_i | i \in \mathcal{V})$ – generators' rotational inertia
- $\gamma = (\gamma_i | i \in \mathcal{V})$ – generator and load response to local system frequency shifts θ_i via damping and speed droop or via frequency dependent loads
- v_i – voltage at the node i (tightly controlled, potentially varying from node to node)

└ Power Grid Dynamics (advanced topics/methods/techniques)

└ Direct Methods: on-line post-fault analysis

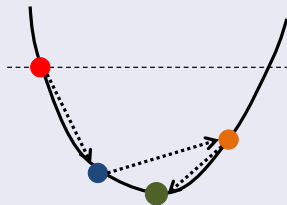
Basic Dynamic Equation [re-cap from Florian lecture]

Re-stated as a Hamiltonian system with damping

$$\dot{\theta}_i = \frac{\partial E(\theta, \varpi)}{\partial \varpi_i}, \quad \dot{\varpi}_i = -\frac{\partial E(\theta, \varpi)}{\partial \theta_i} - \frac{\gamma_i}{M_i} \varpi_i,$$

$$E(\theta; \varpi; v; p) = W + U, \quad W = \sum_{i \in \mathcal{V}} \frac{\varpi_i^2}{2M_i},$$

$$U = \sum_{\{i,j\} \in \mathcal{E}} \beta_{ij} v_i v_j (1 - \cos(\theta_i - \theta_j)) - \sum_{i \in \mathcal{V}} p_i \theta_i$$



- Dynamic side of the “energy function” approach [with a long history in PE]
- $\gamma > 0$: $dE/dt \leq 0$

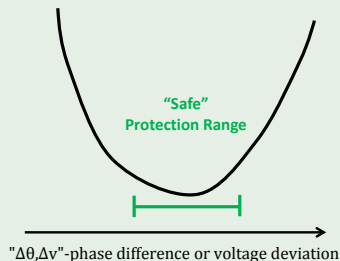
Necessary/Static & Dynamic Synch Conditions

Useful properties of the Energy function

- $\gamma > 0$: $dE/dt \leq 0$
- When $\theta \in \Theta = (\forall_{i,j \in \mathcal{V}} : |\theta_i - \theta_j| \leq \pi/2) \Rightarrow U(\theta; v; p)$ is **convex**.
- Call $\theta_{\min} = \operatorname{argmin}_{\theta \in \Theta} U(\theta, v; p)$ - optimal solution.
 - If θ_{\min} is strictly in the interior of Θ , then θ_{\min} is the only solution of PF within Θ and the dynamics is stable/**synchronizable** in a (possibly small) vicinity of θ_{\min} .
 - If θ_{\min} occurs on the boundary of Θ , then the guarantees of solution existence within Θ are lost.

Distance Protection Model for Relays

- Distance protection relay model is adopted, see e.g. C. Singh and I. Hiskens, Direct assessment of protection operation and non-viable transients, Power Systems, IEEE Transactions on, vol. 16, no. 3, pp. 427434, 2001



At constant voltage reduces to:

- $\theta \in \Theta_{\text{relay}}$, where
$$\Theta_{\text{relay}}(\theta^{\max}) = (\forall \{i, j\} \in \mathcal{E} : |\theta_i - \theta_j| \leq \theta_{ij}^{\max}), \text{ and}$$
$$\theta^{\max} = 2 \arcsin(1/\sqrt{2\beta}).$$
- $\beta = 1.2$ and $\theta^{\max} \approx 1.4$ is the typical choice for zone 2 relays.

└ Power Grid Dynamics (advanced topics/methods/techniques)

└ Direct Methods: on-line post-fault analysis

Prior to Contingency

- System is in a (safe) steady state.
- State estimation is available and reliable (grid is fully visible)
- $t = 0, \forall i \in \mathcal{V} : \dot{\theta}_i = \ddot{\theta}_i = 0$

During (on fault) Dynamics

- Focus on three phase fault (at a generator or load)
- Conduct direct simulations \Rightarrow
- Cauchy problem: $\theta(0) = \theta^{\text{pre}}, \dot{\theta}(0) = 0$
- $[0, \tau_f]$, typically $\tau_f \lesssim 0.5s$
- Aiming to find $\theta^{(\text{post-})} = \theta(\tau_f^-)$ and $\dot{\theta}^{(\text{post-})} = \dot{\theta}(\tau_f^-)$

Post Fault Dynamics

- Initial conditions \Rightarrow
- at all but faulty nodes: $\forall k \in \mathcal{V} \setminus i : \dot{\theta}_k^{(\text{post+})} = \dot{\theta}_k^{(\text{post-})}$
 $\forall k \in \mathcal{V} \setminus i : \theta_k^{(\text{post+})} = \theta_k^{(\text{post-})}$
- at the faulty node:

$$\dot{\theta}_i^{(\text{post+})} = \dot{\theta}_i^{(\text{post-})}$$

$$p_i = \sum_{j: \{i,j\} \in \mathcal{E}} v_i v_j \beta_{ij} \sin(\theta_i^{(\text{post+})} - \theta_j^{(\text{post-})}) \quad \text{fault at a load}$$

$$p_i \tau_f = M_i \left(\dot{\theta}_i^{(\text{post+})} \right)^2 / 2 \quad \text{fault at a generator}$$

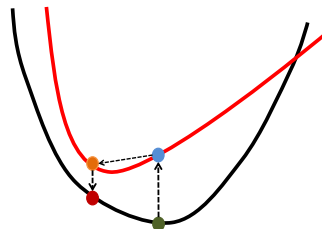
- └ Power Grid Dynamics (advanced topics/methods/techniques)

- └ Direct Methods: on-line post-fault analysis

Prior to Contingency

During (on fault) Dynamics

Post Fault Dynamics

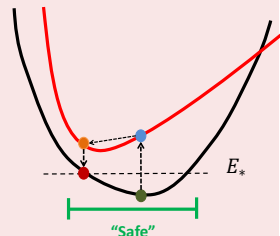


Post-fault Dynamics as a Convex Optimization

- Aim to predict if the dynamics brings the system back to the stable (good) minimum or not
- **without running direct simulations** — instead formulating efficient static optimization scheme
- Use direct simulations to validate results and measure conservatism

Set of Convex Optimization Problems

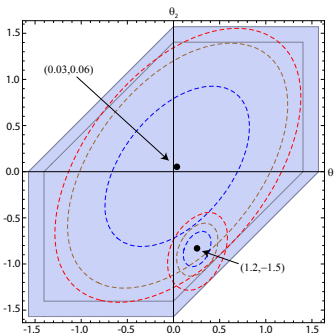
$$\begin{aligned}
 \forall (i, j) \in \mathcal{E} : \quad & \hat{\theta}_{ij} \doteq \arg \max_{\theta} |\theta_i - \theta_j| \\
 \text{s.t.} \quad & U(\theta; v; p) \leq E_* \\
 & E_* \doteq E(\theta^{(\text{post}+)}; \dot{\theta}^{(\text{post}+)}; v; p) \\
 & \theta \in \Theta_{\text{relay}}
 \end{aligned}$$



└ Power Grid Dynamics (advanced topics/methods/techniques)

└ Direct Methods: on-line post-fault analysis

Three Node Illustration



- Two (overlaid) cases shown: $p_1 = 0.03, p_2 = .06$ and $p_1 = 1.2, p_2 = -1.5$
- The gray-blue colored domain shows Θ .
- The gray line bounds the sub-domain Θ_{relay} for $\beta = 1.2$.
- Red and brown dashed lines show iso-lines of the maximum post-fault energy E_{max} that limit the domains of safe recovery for Θ - and Θ_{relay} -constrained systems respectively.
- The values for the cases are
 $E_{\text{min}} \approx 0., E_{\text{max};\Theta} \approx 1.34, E_{\text{max};\text{relay}} \approx 1.1$ and
 $E_{\text{min}} \approx -0.7, E_{\text{max};\Theta} \approx -0.67, E_{\text{max};\text{relay}} \approx -0.63$ respectively.

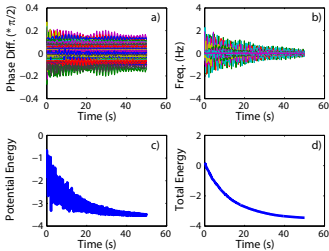
$$U(\theta_1, \theta_2) = (1 - \cos(\theta_1))/0.8 + (1 - \cos(\theta_2))/1.2 + 1 - \cos(\theta_1 - \theta_2) - p_1\theta_1 - p_2\theta_2$$

Numerical Experiment Set Up

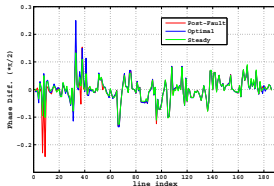
- IEEE 118-bus test system
- E_* is computed from post-fault simulations (small damping is introduced for loads and generators)
- Relay limit is set to $\pi/8$
- We compare **Direct Simulations** with **Efficient Optimization**

- Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis



- Fault duration 0.3 sec at node #9 (the node producing the largest E_* for this duration), $E_* \approx 0$.
- max-phase-difference is close but still smaller than $\pi/8$
- $E_{\min} = -3.56$ steady state value
- However, $E_{\max} = -3.4 - \max E_*$ for given $U(\theta; v, p)$ with guaranteed $|\theta_i - \theta_j| \leq \pi/8, \forall(i, j)$.
- The estimates are conservative.



- Comparison of the post-fault (red), max-energy-optimal (blue), and stationary (green) configurations of phase
- The difference pattern is sparse.

Lessons (for the post-fault story)

- :) **Efficient and provably accurate** method to test post-fault stability, based on the **convex structure** of the energy function, is developed
- :(Experiments show that the bare method is conservative, however it also helps to understand the source of the conservatism

Path Forward – Towards reducing conservatism

- Accounting for proximity to initial conditions — not all boundaries are reachable (too far, wrong angle or insufficient kinetic energy). Possible solution – to remove the max-test for such boundaries from the formulation.
- Observed/desired sparsity of the change (from cleared configuration to potentially achievable during the post-fault dynamics) may be used (?) to reduce conservatism.
- Explore hybrid options merging existing methods/heuristics developed for direct analysis (by H.D. Chung & co-authors, e.g. Controlling UEP + BCU – see the book of H.D. Chung) with the convex optimization idea/formulation

- └ Power Grid Dynamics (advanced topics/methods/techniques)

- └ Modeling Faults and Fast Transients in Distribution

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

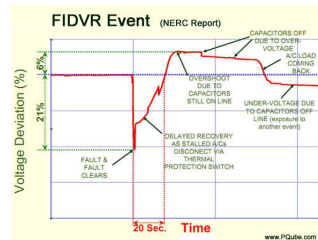
- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

Recorded Distribution/Transmission Voltage Events

- TVA. Blistering Sat of Aug 22, 1987. Cascading Voltage Collapse in West Tennessee. Fault at 115KV switch. Cleared in 1s. Continued into 161KV and 500KV lines for 10-15s. Resulted in the loss of 700MW in Memphis. **Motor loads stalled** and drawn large amount of reactive power even after the fault was cleared.
- 1988 event in Florida reported in "Air Conditioner Respond to Transmission Fault" by J. W. Shaffer in 1997 ... "In the last ten years there have been at least eight events in which normally cleared (**in 2-3 cycles**) multi-phase events in Southern Florida have caused a significant drop in customer load (200-825MW)."
- 1990 Egypt ... 1999 metro area Atlanta, Arizona, Southern California ... NERC Planning Committee White Paper on "Fault Induced Delayed Voltage Recovery" by **Transmission** Issues Subcommittee

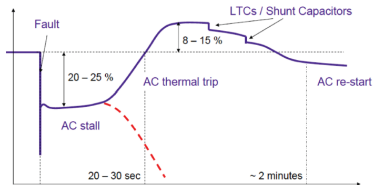
- delays (between cause and the result)
- nonlinearity of loads plays a significant role
- many inductive motors **simultaneously** affected
- initiated (fault) at the **transmission-to-distribution** interface, matures within **distribution**, cascades into **transmission**



Typical FIDVR Following a 230-kV Transmission Fault in Southern California

Modeling extended FIDVR

C.Duclut, S.Backhaus & MC (PRE '12)



courtesy of D. Kostyrev and B. Lesieutre

- Observed in feeders with many induction motors (air-conditioning)
- Uncontrolled depressed voltage can spread causing a larger outage
- Hypothesis (Hiskens, Lesieutre, Chassin, ...): the events are caused by many air conditioners stalled
- Modeling the event is a challenge

DBC '12 Contribution - Modeling of FIDVR over extended feeder

- Observation (simulations – consistent with measurements): soliton-like propagation of “stalled” phase/front
- Coarse-grained (reduced) PDE modeling of the “extended” FIDVR
- Extended recently to account for effects of disorder (“frozen” parameters irregularities) – [I. Stolbova. SB, MC '14]

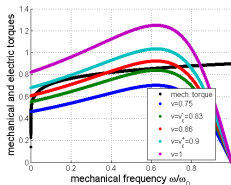
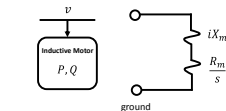
└ Power Grid Dynamics (advanced topics/methods/techniques)

└ Modeling Faults and Fast Transients in Distribution

Modeling Individual Motor

Popovic, Hiskens, Hill '98

minimal model of the motor



$$P = \frac{sR_m v^2}{R_m^2 + s^2 X_m^2}$$

$$Q = \frac{s^2 X_m v^2}{R_m^2 + s^2 X_m^2}$$

$$M \frac{d}{dt} \omega = \frac{P}{\omega_0} - T_0 (\omega/\omega_0)^\alpha \quad (\text{dynamics})$$

$$s = 1 - \omega/\omega_0$$

s is the slip against the base frequency)

v is the voltage at the motor

P, Q are real and reactive power consumed by the motor

T_0, α torque constant and scaling coefficient

R_m, X_m resistance and inductance of the motor

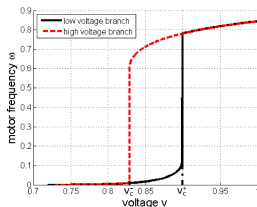
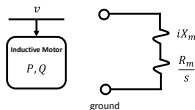
Explanation for "lumped" FIDVR

- Hysteresis: The motor is trapped in the stalled (low-voltage) state!
- First order phase transition. Bifurcation (stability). Spinodal points.

Modeling Individual Motor

Popovic, Hiskens, Hill '98

minimal model of the motor



$$P = \frac{s R_m v^2}{R_m^2 + s^2 X_m^2}$$

$$Q = \frac{s^2 X_m v^2}{R_m^2 + s^2 X_m^2}$$

$$M \frac{d}{dt} \omega = \frac{P}{\omega_0} - T_0 (\omega / \omega_0)^\alpha \quad (\text{dynamics})$$

$$s = 1 - \omega / \omega_0$$

s is the slip against the base frequency)

v is the voltage at the motor

P, Q are real and reactive power consumed by the motor

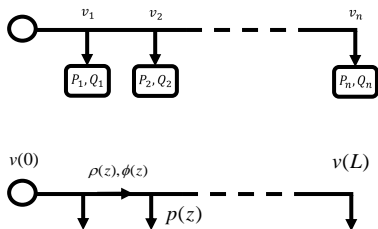
T_0, α torque constant and scaling coefficient

R_m, X_m resistance and inductance of the motor

Explanation for "lumped" FIDVR

- Hysteresis: The motor is trapped in the stalled (low-voltage) state!
- First order phase transition. Bifurcation (stability). Spinodal points.

Feeder with Many (distributed) Inductive Motors



- Spatially-continuous version of Dist.Flow [Baran, Wu (1989)]

$$\partial_z \rho = -p - r \frac{\rho^2 + \phi^2}{v^2}$$

$$\partial_z \phi = -q - x \frac{\rho^2 + \phi^2}{v^2}$$

$$v \partial_z v = -(r\rho + x\phi)$$

$$p = \frac{sr_m v^2}{r_m^2 + s^2 x_m^2}$$

$$q = \frac{s^2 x_m v^2}{r_m^2 + s^2 x_m^2}$$

$$\mu \frac{d}{dt} \omega = \frac{p}{\omega_0} - \tau_0 \left(\frac{\omega}{\omega_0} \right)^\alpha$$

$$v(0) = 1, \rho(L) = \phi(L) = 0$$

Reduced model of the “extended” feeder

Easy to analyze dynamics: PDE.

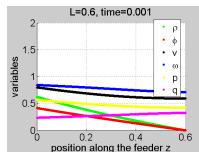
└ Power Grid Dynamics (advanced topics/methods/techniques)

└ Modeling Faults and Fast Transients in Distribution

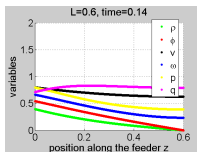
Dynamics/Transitions in an Extended Feeder (I)

Example of a Large Fault → feeder is stalled

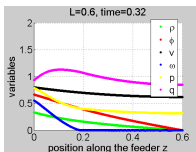
(Movie Large Fault)



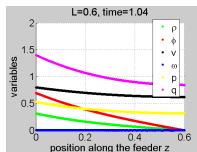
(a)



(b)



(c)



(d)

■ (a) Pre fault

■ (b) Immediately past fault

■ (c) Later in the process

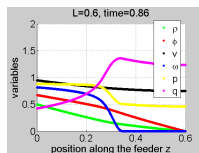
■ (d) The feeder is fully stalled

└ Power Grid Dynamics (advanced topics/methods/techniques)

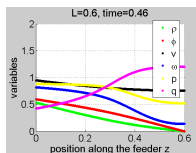
└ Modeling Faults and Fast Transients in Distribution

Dynamics/Transitions in an Extended Feeder (II)

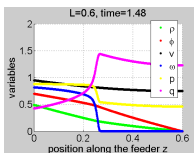
Example of a Small Fault → feeder is partially stalled (Movie Small Fault)



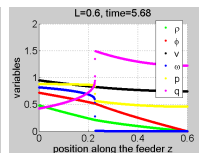
(a)



(b)



(c)



(d)

- (a) Immediately past fault
- (b) Later in the process

- (c) Front advances
- (d) Stabilized, part. stalled

► dynamics of restoration

Re-cap of the FIDVR story

- The 1+1 (space+time) continuous model of distribution
- Integrating multiple bi-stable individual motors into power flow
- Emergence of multiple spatially-extended states/transitions

Conclusions Drawn from Experiments/Numerics concern

- Hysteresis
- Self-Similar Transients

... relevant research (done or work in progress) ...

- Inhomogeneity (disorder), stochasticity (noise): what is the probability that the feeder with a given level of disorder will recover?
- Effects of other devices, e.g. **distributed generation and control** (PV) ...
- Possible **cascade** – from feeder to feeder (within substation) ... to transmission
- ... more theory, e.g. resolving phase transition boundaries, propagation of soliton-like phase fronts, etc
- ... reduced model of FIDVR for transmission studies ... resolving dynamics faithfully

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

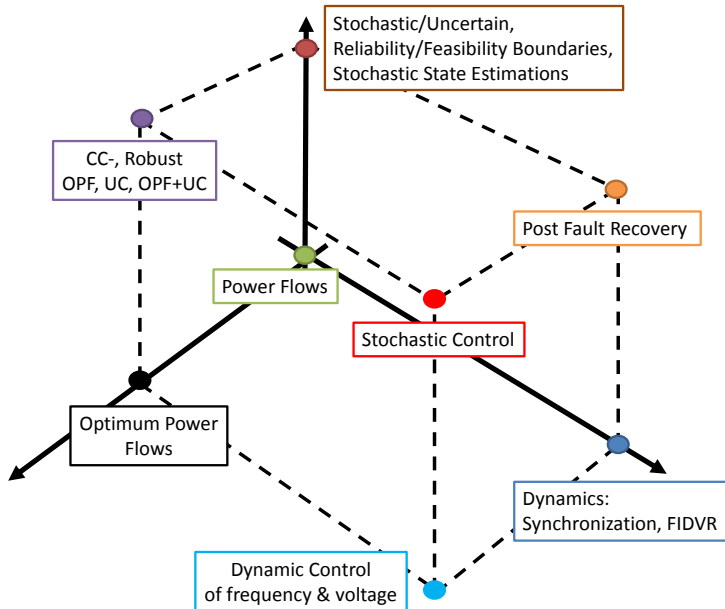
- Optimum Gas Flows

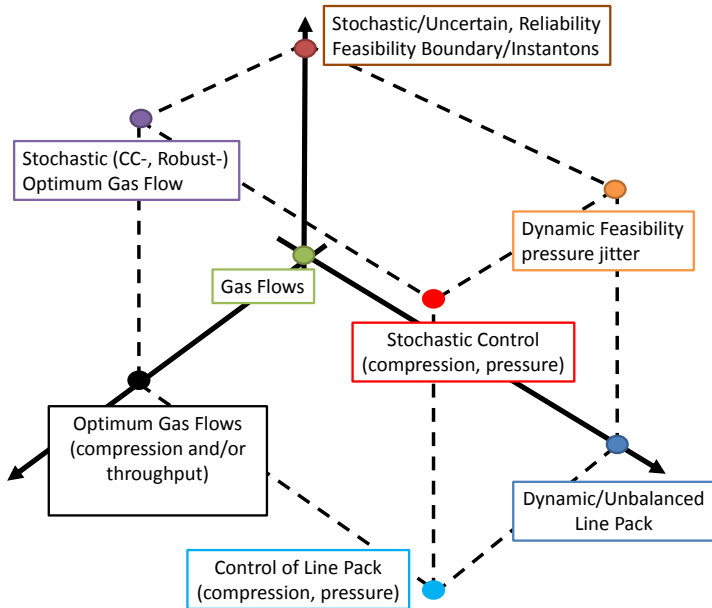
Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges





- └ Gas Flows: Dynamic, Static, Optimization

- └ Gas Dynamics – pipeline fundamentals

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals**

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

2 min crash course on the hydro (gas) dynamics

- single pipe; not tilted (gravity is ignored); constant temperature
- ideal gas, $p \sim \rho$ – pressure and density are in a linear relation
- all fast transients are ignored – gas flow velocity is significantly slower than the speed of sound, $u \ll c_s$
- turbulence is modeled through turbulent friction; mass flow, $\phi = u\rho$, are averaged across the pipe's cross-section

$$\left. \begin{array}{l} \underbrace{\partial_t \rho + \partial_x(u\rho) = 0}_{\text{conservation of mass}} \\ \underbrace{\partial_t(\rho u) + \partial_x(\rho u^2) + \partial_x p = -\frac{\rho u |u|}{2D} f}_{\text{conservation of energy}} \end{array} \right\} \approx \Rightarrow \left\{ \begin{array}{l} \underbrace{c_s^{-2} \partial_t p + \partial_x \phi = 0}_{\text{conservation of mass}} \\ \underbrace{\partial_x p^2 + \frac{\beta}{D} \phi |\phi| = 0}_{\text{conservation of energy}} \end{array} \right.$$

Approximations ... allowing to resolve flows analytically (lamp description)

Stationary, balanced regime [standard]

$$\phi = \text{const}, \quad p_{in}^2 - (p(x))^2 = x\beta\phi|\phi|/D$$

Unbalanced, linearized line-pack [non-standard]

$$\phi = \phi_{\text{st}}(x) + \delta\phi(t, x), \quad p = p_{\text{st}}(x) + \delta p(t, x)$$

- └ Gas Flows: Dynamic, Static, Optimization

- └ Static/Balanced Flows. Compression. Energy Function.

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

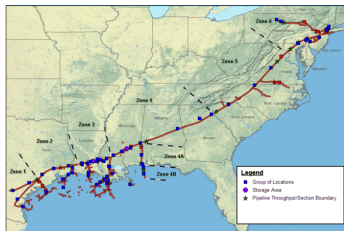
- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

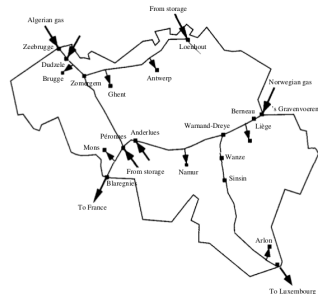
└ Gas Flows: Dynamic, Static, Optimization

└ Static/Balanced Flows. Compression. Energy Function.

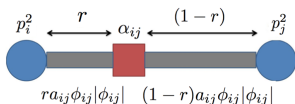
Gas Flows. Steady (balanced) Case.



Belgian gas network.



without compressors, $\alpha_{ij} = 1$

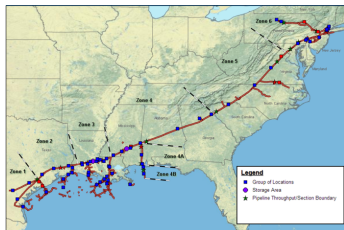


- Gas Flow Equations: $(\sum_i q_i = 0, \quad a_{ij} = L_{ij}\beta_{ij}/D_{ij})$
 $\forall (i,j) : \quad p_i^2 - p_j^2 = a_{ij}\phi_{ij}^2$
 $\forall i : \quad q_i = \sum_{j:(i,j) \in \mathcal{E}} \phi_{ij} - \sum_{j:(j,i) \in \mathcal{E}} \phi_{ji}$

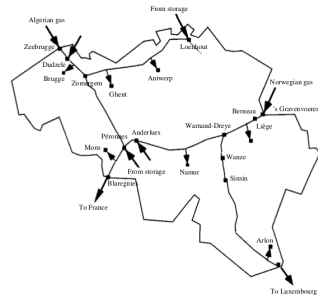
└ Gas Flows: Dynamic, Static, Optimization

└ Static/Balanced Flows. Compression. Energy Function.

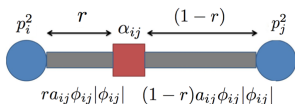
Gas Flows. Steady (balanced) Case.



Belgian gas network.



with compressors, $\alpha_{ij} \leq 1$



- Gas Flow Equations: ($\sum_i q_i = 0$, $a_{ij} = L_{ij}\beta_{ij}/D_{ij}$)

$$\forall(i,j) : \quad \alpha_{ij}^2 = \frac{p_j^2 + (1-r)a_{ij}\phi_{ij}^2}{p_i^2 - ra_{ij}\phi_{ij}|\phi_{ij}|}$$

$$\forall i : \quad q_i = \sum_{j:(i,j) \in \mathcal{E}} \phi_{ij} - \sum_{j:(j,i) \in \mathcal{E}} \phi_{ji}$$

└ Gas Flows: Dynamic, Static, Optimization

└ Static/Balanced Flows. Compression. Energy Function.

Energy Function Formulations of Gas Flow Eqs.

Gas Flows \Leftarrow Minimization of Looses (turbulent friction)

$$\begin{aligned} & \min_{\phi} \quad \overbrace{\frac{1}{3} \sum_{(i,j)} a_{ij} |\phi_{ij}|^3}^{\text{Looses in pipes}} \\ & \text{s.t.} \quad \forall i : \quad \overbrace{q_i = \sum_{j:(i,j)} \phi_{ij}}^{\text{nodal flow balance}} \end{aligned}$$

- The optimization is convex
- p_i^2 - are Lagrangian multipliers
- Dual formulation \Rightarrow stated as optimization over nodal potentials (pressures)
- Generalization for the case of additive compression is straightforward
- Generalization for the case of multiplicative compression is **still a challenge**

- J. J. Maugis, RAIRO Recherche Opérationnelle/Operations Research, 11(2):243248, 1977
- Most recently used in — F. Babonneau, Y. Nesterov and J.-P.Vial, Operations Research Operations Research, 60 (1): 34-47, 2012 (for two-level optimization)

- └ Gas Flows: Dynamic, Static, Optimization

- └ Dynamics. Line Pack. Approximations.

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

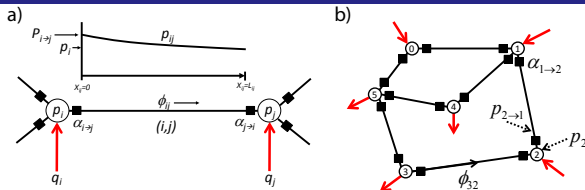
- Optimum Gas Flows

Gas-Grid Reliability [connecting to “stochastic” Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges



Basic Dynamic Equations (PDEs) – fast transients are ignored

$$\forall t \in [0, \tau], \quad \forall \{i, j\} \in \mathcal{E}, \quad \forall x \in [0; L_{ij}] :$$

$$c_s^{-2} \partial_t p_{ij}(t, x) + \partial_x \phi_{ij}(t, x) = 0$$

$$\partial_x p_{ij}(t, x) + \frac{\beta}{2d} \frac{\phi_{ij}(t, x) |\phi_{ij}(t, x)|}{p_{ij}(t, x)} = 0$$

Initial/Boundary Conditions

(an example)

$$\forall t \in [0, \tau], \quad \forall (i, j) \in \mathcal{E} : \quad p_{ij}(t, 0) = p_{i \rightarrow j}(t)$$

$$p_{ij}(t, L_{ij}) = p_{j \rightarrow i}(t), \quad p_{i \rightarrow j} = p_i \alpha_{i \rightarrow j}, \quad p_{j \rightarrow i} = p_j \alpha_{j \rightarrow i}$$

$$\forall t \in [0, \tau], \quad \forall i \in \mathcal{V} : \quad \sum_{j: (i,j) \in \mathcal{E}} \phi_{ij}(t, 0) = q_i(t)$$

$$\forall \{i, j\} \in \mathcal{E}, \quad \forall x_{ij} \in [0, L_{ij}] : \quad \phi_{ij}(0; x_{ij}) = \phi_{ij}^{(in)}(x_{ij}), \quad p_{i=0}(t=0) = p_0$$

DNS for PDEs – [in collaboration with S. Dyachenko (UA), A. Korotkevich (UNM)]

- Line Pack = the system is not balanced, $\sum_i q_i(t) \neq 0$

- Illustration for a long line with/without compressor

Case A

- Flux is constant at the entrance, oscillates at the exit

Case B

- Pressure is constant on the entrance (slack bus), flux oscillates at the exit

Case C

- Fluxes are constant, but different (unbalanced), at the entrance and at the exit

- Four segments/pipes with/without (after the 2 segment)
- Inflow is fixed at the entrance. Injection/consumptions oscillates at four nodes. Constant in average (over time) but not at any instance.

Compression in the middle**No compression**

Linear approximation [around steady/balanced solution]

- $q(t) = q^{(\text{st})} + \xi(t)$, $p(t) = p^{(\text{st})} + \delta p(t)$, $\phi(t) = \phi^{(\text{st})} + \delta \phi(t)$
- $\xi \ll q^{(\text{st})}$, $\delta p \ll p^{(\text{st})}$, $\delta \phi \ll \phi^{(\text{st})}$

$$\forall t \in [0, \tau], \quad \forall \{i, j\} \in \mathcal{E}, \quad \forall x \in [0; L_{ij}] :$$

$$c_s^{-2} \partial_t \delta p_{ij} + \partial_x \delta \phi_{ij} = 0$$

$$\partial_x \delta p_{ij} + \frac{\beta}{2d} \left(\frac{\delta \phi_{ij} |\phi_{ij}^{(\text{st})}|}{p_{ij}^{(\text{st})}} + \frac{\phi_{ij}^{(\text{st})} |\delta \phi_{ij}|}{p_{ij}^{(\text{st})}} - \frac{\delta p_{ij} \phi_{ij}^{(\text{st})} |\phi_{ij}^{(\text{st})}|}{(p_{ij}^{(\text{st})})^2} \right) = 0$$

$$\forall t \in [0, \tau], \quad \forall (i, j) \in \mathcal{E} : \quad \delta p_{i \rightarrow j} = \delta p_{i \alpha_{i \rightarrow j}}$$

$$\delta p_{ij}(t, 0) = \delta p_{i \rightarrow j}(t), \quad \delta p_{ij}(t, L_{ij}) = \delta p_{j \rightarrow i}(t)$$

$$\forall t \in [0, \tau], \quad \forall i \in \mathcal{V} : \quad \sum_{j: (i, j) \in \mathcal{E}} \delta \phi_{ij}(t, 0) = \xi_i(t)$$

$$\delta p_{ij} = a_{ij}(t) Z_{ij}(x) + b_{ij}(t, x),$$

$$\partial_x Z_{ij} - \frac{\beta}{d} \frac{\phi_{ij}^{(\text{st})} |\phi_{ij}^{(\text{st})}|}{p_{ij}^{(\text{st})}} Z_{ij} = 0$$

$$b_{ij}(t, x) \ll a_{ij}(t) Z_{ij}(x) \Rightarrow \text{at } t \gg t_0$$

- t_0 is the typical time when the balance (in flows) is restored = correlation time of $\xi(t)$

- Pressure deviation (from the steady solution) — jitter, grow in time diffusively \Rightarrow stochastic consequences to be discussed latter

└ Gas Flows: Dynamic, Static, Optimization

└ Dynamics. Line Pack. Approximations.

Adiabatic approximation

- Consider example of a single pipe
- Choose boundary conditions, e.g. pressures at the two ends are fixed
- $t \gg L/c_s$, looking for slow dynamics, e.g. $\dot{p}t/p \ll 1$

$$p = P + \delta p, \quad P = \sqrt{p_1^2 - (p_1^2 - p_2^2)x/L}$$

$$\phi = \Phi + \delta \phi, \quad \Phi^2 = \frac{d(p_1^2 - p_2^2)}{\beta L}$$

$$c_s^{-2} \partial_t p + \partial_x \phi = 0$$

$$\partial_x p^2 + \frac{\beta}{d} \phi |\phi| = 0$$

$$p(t, x=0) = p_1(t), \quad p(t, x=L) = p_2(t)$$

$$\Rightarrow \delta \phi = \frac{2L}{3c_s^2} \left[\frac{(p_1^2 - (p_1^2 - p_2^2)x/L)^{3/2}}{p_1^2 - p_2^2} - \frac{2(p_1^5 - p_2^5)}{5(p_1^2 - p_2^2)^2} \right]_t$$

- $p_{1,2}$ are (slow) time-dependent

$$\delta \phi_1 = \frac{2L}{15c_s^2} \left[\frac{3p_1^3 + 6p_1^2 p_2 + 4p_1 p_2^2 + 2p_2^3}{(p_1 + p_2)^2} \right]_t$$

$$\delta \phi_2 = -\frac{2L}{15c_s^2} \left[\frac{3p_2^3 + 6p_2^2 p_1 + 4p_2 p_1^2 + 2p_1^3}{(p_1 + p_2)^2} \right]_t$$

- The resulting equations/rerelations are time dependent ... but nodal (!!)-ODEs not (!!)-PDEs

- Suggested by M. Herty, et al (2008)

Optimum Gas Flow (OGF)

Minimizing the cost of compression (\sim work applied externally to compress)

$$\min_{\alpha, p} \sum_{(i,j)} \frac{c_{ij} \phi_{ij}}{\eta_{ij}} \left(\alpha_{ij}^m - 1 \right)^+ \quad \left| \quad \begin{aligned} \forall (i,j) : \quad \alpha_{ij}^2 &= \frac{p_j^2 + (1-r) a_{ij} \phi_{ij}^2}{p_i^2 - r a_{ij} \phi_{ij}^2} \\ \forall i : \quad 0 \leq \underline{p}_i &\leq p_i \leq \bar{p}_i \\ \forall (i,j) : \alpha_{ij} &\leq \bar{\alpha}_{ij} \end{aligned} \right.$$

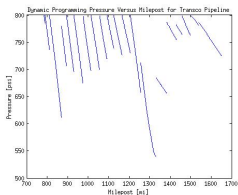
- $0 < m = (\gamma - 1)/\gamma < 1$, γ - gas heat capacity ratio (thermodynamics)

- The problem is **convex on trees** (many existing gas transmission systems are trees) \Leftarrow through **Geometric Programming** (log-function transformation)

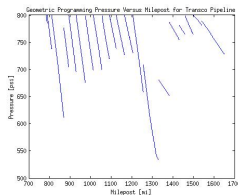
- **S. Misra, M. W. Fisher, S. Backhaus, R. Bent, MC, F. Pan**, *Optimal compression in natural gas networks: a geometric programming approach*, IEEE Transaction on Network Controls (CONES), Nov 2014, arXiv:1312.2668

OGF experiments (Transco pipeline)

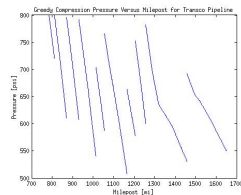
Dynamic Programming of (Wong, Larson '68)



Geometric Programming (ours)



Greedy Compression (current practice)



GP is advantageous over DP

- Exact = no-need to discretize.
- Faster. Allows distributed (ADMM) implementation.
- Convexity is lost in the loopy case. However, an efficient heuristics is available. [work in progress]
- This is only one of many possible OGF formulations. Another (Norwegian/European) example – maximize throughput.
- Major handicap of the formulation (ok for scheduling but) = did not account for the **line pack** (dynamics/storage in lines for hours)

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to "stochastic" Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

- Gas-Grid Coupling, Challenges

Probabilistic Forecast of the Pressure (Gas System) Jitter

Explicit expression for pressure fluctuations via in/out flows [from linear approximation]

$$\forall t, \quad \forall \{i, j\} \in \mathcal{E}, \forall x \in [0, L_{ij}] :$$

$$\delta p_{ij}(t, x) \approx \frac{c_s^2 \Xi(t)}{\sum_{\{i, j\} \in \mathcal{E}} c_{ij}} c_{ij} Z_{ij}(x), \quad \Xi(t) \doteq \int_0^t dt' \sum_{i \in \mathcal{V}} \xi_i(t')$$

$$\forall i, \quad \forall j, k \text{ s.t. } (i, j), (i, k) \in \mathcal{E} : \quad \frac{c_{ij} Z_{ij}(0)}{\alpha_{i \rightarrow j}} = \frac{c_{ik} Z_{ik}(0)}{\alpha_{i \rightarrow k}}$$

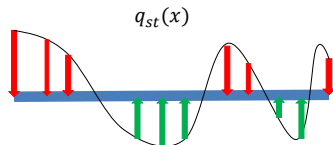
Universal & Network- Inhomogeneous Diffusive Jitter

$$\mathcal{P}(\delta p_{ij}(t, x) = \delta) \rightarrow (2\pi t D_{ij}(x))^{-1/2} \exp\left(-\frac{\delta^2}{2t D_{ij}(x)}\right)$$

$$D_{ij} = \left(\frac{c_s^2 c_{ij} Z_{ij}(x)}{\sum_{\{k, l\} \in \mathcal{E}} c_{kl}} \right)^2 \left\langle \left(\sum_{n \in \mathcal{V}} \xi_n(t') \right)^2 \right\rangle$$

- Averaging over random in/out fluctuations
- Asymptotic (Law of Large Numbers) Gaussianity
- Covariance of pressure fluctuations grows linearly with time
- Spatial (sensitive to steady solution) and temporal (sensitive to in/out flow fluctuations) are separated

Pressure Jitter — example of 1d system



Steady (balanced) continuous profile
of gas injection/consumption

- $q(t, x) = q_{st}(x) + \xi(t, x), \quad \xi(t, x) \ll q_{st}(x)$
 - $q_{st}(x)$ is the forecasted consumption/injection of gas
 - $\xi(t, x)$ actual fluctuating/random profile of consumption/injection, e.g. **fluctuations due to gas power plants following wind turbines**

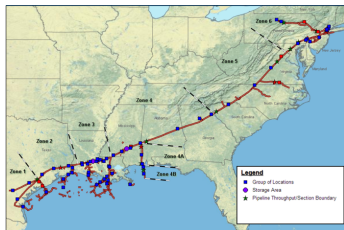
One dimensional (1+1) model – distributed injection/consumption and compression

- mass balance:
$$c_s^{-2} \partial_t p + \partial_x \phi = -q(t, x)$$
 - energy balance:
$$\partial_x p + \frac{\beta}{2d} \frac{\phi |\phi|}{p} = \gamma(x) p$$
 - $\gamma(x)$ – distributed compression – assumed known
- generalized to an arbitrary graph
 - **S. Backhaus, MC, and V. Lebedev, arXiv:1411.2111**
 - validated against DNS – to be published; in collaboration with S. Dyachenko (UA), A. Korotkevich (UNM)

└ Gas-Grid Reliability [connecting to "stochastic" Wed]

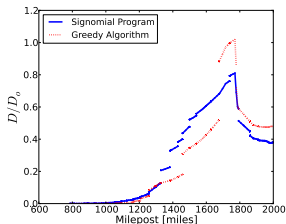
└ Probabilistic State Estimations – a Gas Example

Transco Pipeline Illustrations



- Transco data - available online at <http://www.1line.williams.com/Transco/index.html>
- 24 hours period on Dec 27, 2012;
 $\phi_0 \approx 20 \text{ kg/s}$; ≈ 70 nodes; pressure range 500 – 800 *psi*
- mile post 1771 (large load in NJ, NYC)
- Marcellus shell (mile post 2000)
- mile post 1339 (large load in NC)

Diffusion coefficient as a function of distance along the Transco mainline

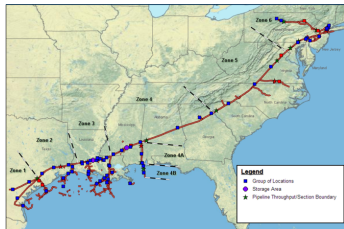


- peak at milepost 1771
- peak is much higher for the greedy case

└ Gas-Grid Reliability [connecting to "stochastic" Wed]

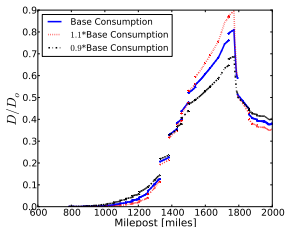
└ Probabilistic State Estimations – a Gas Example

Transco Pipeline Illustrations



- Transco data - available online at <http://www.1line.williams.com/Transco/index.html>
- 24 hours period on Dec 27, 2012;
 $\phi_0 \approx 20 \text{ kg/s}$; ≈ 70 nodes; pressure range 500 – 800psi
- mile post 1771 (large load in NJ, NYC)
- Marcellus shell (mile post 2000)
- mile post 1339 (large load in NC)

Diffusion coefficient as a function of distance along the Transco mainline

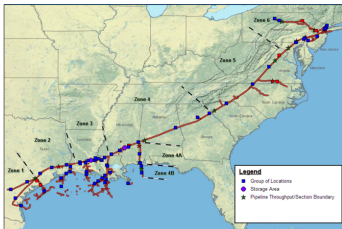


- global consumption and injection re-scaled by a uniform factor
- peak remains at the milepost 1771
- observe larger pressure fluctuations for larger system loads

└ Gas-Grid Reliability [connecting to "stochastic" Wed]

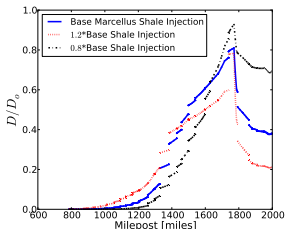
└ Probabilistic State Estimations – a Gas Example

Transco Pipeline Illustrations



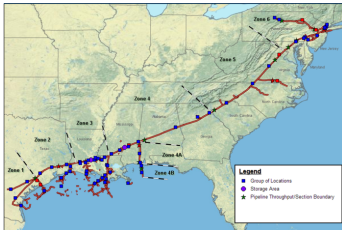
- Transco data - available online at <http://www.1line.williams.com/Transco/index.html>
- 24 hours period on Dec 27, 2012;
 $\phi_0 \approx 20 \text{ kg/s}$; ≈ 70 nodes; pressure range 500 – 800 psi
- mile post 1771 (large load in NJ, NYC)
- Marcellus shell (mile post 2000)
- mile post 1339 (large load in NC)

Diffusion coefficient as a function of distance along the Transco mainline



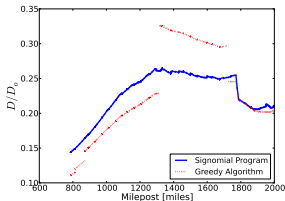
- Marcellus Shale injections scaled up by a factor and the corresponding amount of injections removed from the Gulf.
- still a peak at milepost 1771. Higher scaling factors result in smaller pressure fluctuations, especially noticeable at the Marcellus Shale.

Transco Pipeline Illustrations



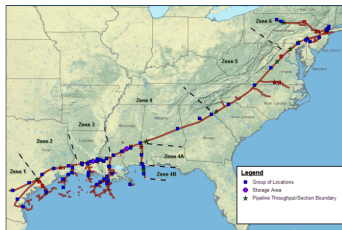
- Transco data - available online at <http://www.1line.williams.com/Transco/index.html>
- 24 hours period on Dec 27, 2012;
 $\phi_0 \approx 20 \text{ kg/s}$; ≈ 70 nodes; pressure range 500 – 800 psi
- mile post 1771 (large load in NJ, NYC)
- Marcellus shell (mile post 2000)
- mile post 1339 (large load in NC)

Diffusion coefficient as a function of distance along the Transco mainline



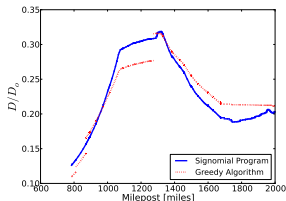
- Load redistributed from the large load in NYC to the Gulf and Marcellus Shale, leaving the large load in NJ unaltered.
- This causes the appearance of a new global maximum at milepost 1319 which is the location of a large load in NC.
- Since the NJ load was not redistributed, a local maximum remains at milepost 1771.

Transco Pipeline Illustrations



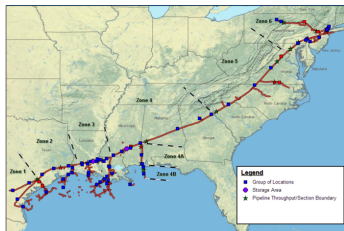
- Transco data - available online at <http://www.1line.williams.com/Transco/index.html>
- 24 hours period on Dec 27, 2012;
 $\phi_0 \approx 20 \text{ kg/s}$; ≈ 70 nodes; pressure range 500 – 800 psi
- mile post 1771 (large load in NJ, NYC)
- Marcellus shell (mile post 2000)
- mile post 1339 (large load in NC)

Diffusion coefficient as a function of distance along the Transco mainline



- load redistributed from the large loads in NYC and NJ closer to the Gulf and Marcellus Shale.
- The global maximum at milepost 1319 remains, but the local maximum at milepost 1771 disappears since the large load has been removed from this area.

Transco Pipeline Illustrations



- Transco data - available online at <http://www.1line.williams.com/Transco/index.html>
- 24 hours period on Dec 27, 2012;
 $\phi_0 \approx 20 \text{ kg/s}$; ≈ 70 nodes; pressure range 500 – 800 *psi*
- mile post 1771 (large load in NJ, NYC)
- Marcellus shell (mile post 2000)
- mile post 1339 (large load in NC)

Diffusion coefficient as a function of distance along the Transco mainline

Main point (qualitative):

- change in the base (steady/forecasted) solution has a profound effect on the pressure fluctuations

- └ Gas-Grid Reliability [connecting to "stochastic" Wed]

- └ Distance to Failures. Instantons – a Grid Example.

Outline

Power Flows (advanced topics/methods/techniques)

- Energy Function

- Distribution Flows

- Linear Approximations beyond DC (coupled & decoupled)

Power Grid Dynamics (advanced topics/methods/techniques)

- Direct Methods: on-line post-fault analysis

- Modeling Faults and Fast Transients in Distribution

Gas Flows: Dynamic, Static, Optimization

- Gas Dynamics – pipeline fundamentals

- Static/Balanced Flows. Compression. Energy Function.

- Dynamics. Line Pack. Approximations.

- Optimum Gas Flows

Gas-Grid Reliability [connecting to "stochastic" Wed]

- Probabilistic State Estimations – a Gas Example

- Distance to Failures. Instantons – a Grid Example.

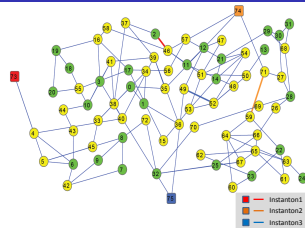
- Gas-Grid Coupling, Challenges

└ Gas-Grid Reliability [connecting to "stochastic" Wed]

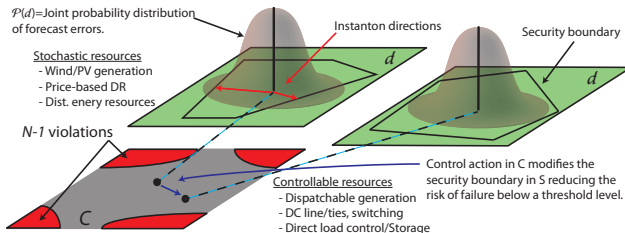
└ Distance to Failures. Instantons – a Grid Example.

Reliability Measure of Power System Under Uncertainty

- Stochastic/uncontrollable participants (e.g. renewables) fluctuate
- Just the standard "N-1"-security gives no guarantees under uncertainty



Instantons in Power Systems: MC, F. Pan, M. Stepanov (2010); MC, FP, MS, R. Baldick (2011); S.S. Baghsorkhi, I. Hiskens (2012)



└ Gas-Grid Reliability [connecting to "stochastic" Wed]

└ Distance to Failures. Instantons – a Grid Example.

- How to estimate a probability of a failure?
- How to predict (anticipate) and then prevent the system from going towards a failure?
- Phase space of possibilities is huge (finding the needle in the haystack)

© Original Artist

Reproduction rights obtainable from
www.CartoonStock.com



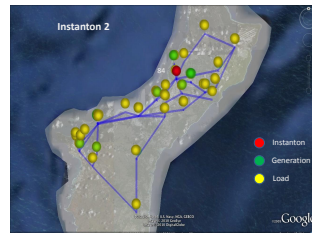
Ed was unlucky enough to find
the needle in the haystack!

© Original Artist

Reproduction rights obtainable from
www.CartoonStock.com



You were right: There's a needle in this haystack...



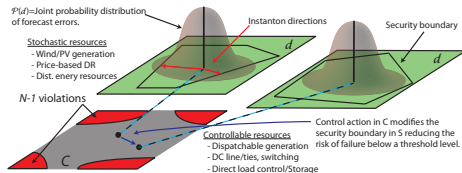
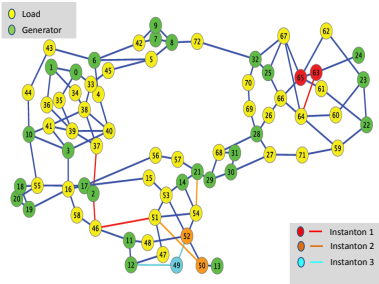
- This type of problems were posed & solved (theoretically & algorithmically) in other disciplines, e.g. Statistical Physics, Error-Corrections, etc.
- We are extending the Methodology **to Energy Systems**

└ Gas-Grid Reliability [connecting to "stochastic" Wed]

└ Distance to Failures. Instantons – a Grid Example.

Extreme Statistics of Failures. Graded List of Instantons.

- RTS 96. DC approx. Full Generation re-dispatch.
- Statistics of forecasted errors is known: $\mathcal{P}(\mathbf{d})$.
- $\mathbf{d} \in \text{SAT}$ = No Power Flow or Generation Violations. SAT is a polytope (may be large).
- $\arg \max_{\mathbf{d}} \mathcal{P}(\mathbf{d}) |_{\mathbf{d} \notin \text{SAT}}$ - most probable instanton



- Efficient optimization heuristics (amoeba search)
- instantons are localized (sparse)
- long correlations
- paradoxes (lowering demand may lead to infeasibility)

Towards a GOOD fluctuations aware optimization/control

- Uncontrollable participants (e.g. renewables) fluctuate
- Standard "N-1"-security gives no guarantees under uncertainty
- First: given statistics of "errors" quantify Probabilistic Distance to Failure = instantons today's example
- Then, account for the probabilistic "errors" and modify existing optimization/control schemes = CC-OPF (D. Bienstock talk at the conference)

└ Gas-Grid Reliability [connecting to “stochastic” Wed]

└ Distance to Failures. Instantons – a Grid Example.

Where are we now? [Instantons — State of the Art]

- Implemented for line overloads and DC power flows
- Demonstrated with both **optimal re-dispatch** and **droop+AGC**
- Demonstrated tractable for realistic networks

Where do/can we go now?

Full AC power flow ... in steps

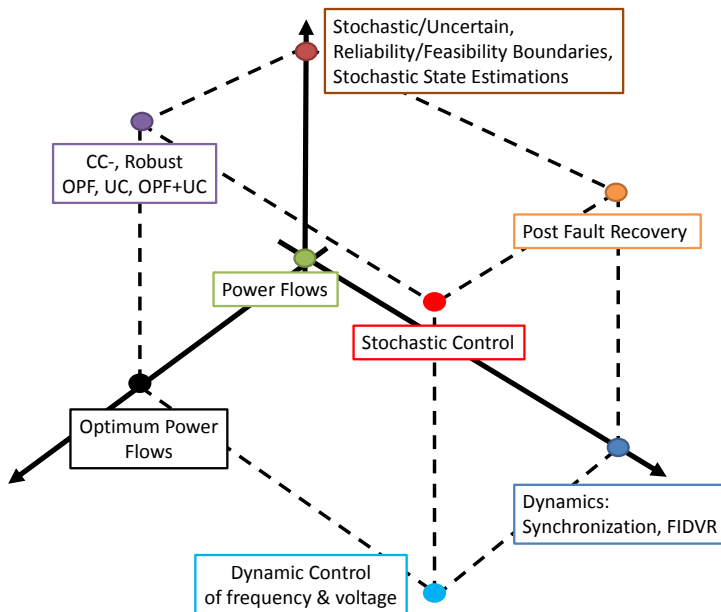
- Line overloads, Linear Coupled and Linear Decoupled (accounting for voltages) — in progress [with J. Kersulis, I. Hiskens, S. Backhaus]
- Line overloads=overheats, dynamics/temperature instantons — in progress [with J. Kersulis, I. Hiskens]
- Loss of synchrony and Voltage Collapse ... through the Energy Function approach – in progress [with D. Dvijotham and S. Low – see Dj talk]

... and beyond

- Incorporation of new/different controls (FACTS, etc)
- Combining instantons with N-1 conditions (for all N-1 possible contingency networks)
- Multiple time frames, changing forecasts [see e.g. posters/work of Y. Dvorkin and M. Lubin, also in the lectures of D. Bienstock]
- Adapt instantons for planning expansion of stochastic grids
- Instantons for individual market participants [work in progress with S. Misra and A. Rudkevich]
- ... a lot of synergy with other “stochastic” methods ... see e.g. lectures of D. Bienstock, A. Conejo, K. Turitsyn and D. Callaway

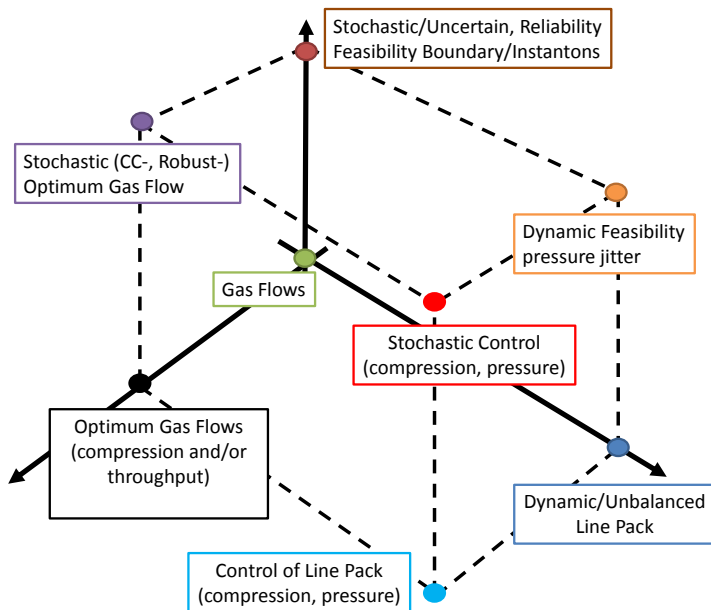
- Gas-Grid Reliability [connecting to "stochastic" Wed]

- Gas-Grid Coupling, Challenges



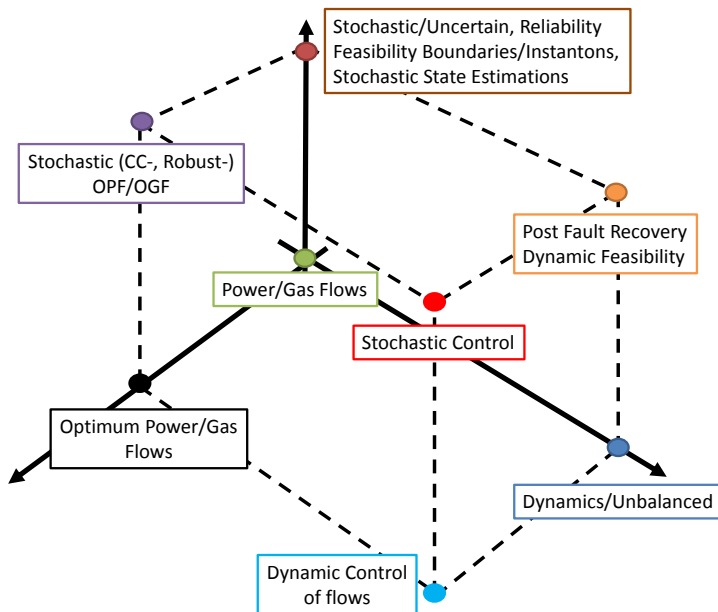
- Gas-Grid Reliability [connecting to "stochastic" Wed]

- Gas-Grid Coupling, Challenges



- Gas-Grid Reliability [connecting to "stochastic" Wed]

- Gas-Grid Coupling, Challenges



└ Gas-Grid Reliability [connecting to "stochastic" Wed]

└ Gas-Grid Coupling, Challenges

Coupled Infrastructures (+ comm.)
=4th dimension

Stochastic/Uncertain, Reliability
Feasibility Boundary/Instantons

CC-, Robust
OPF/OGF

Post Fault Recovery
Dynamic Feasibility

Power/Gas Flows

Stochastic Control

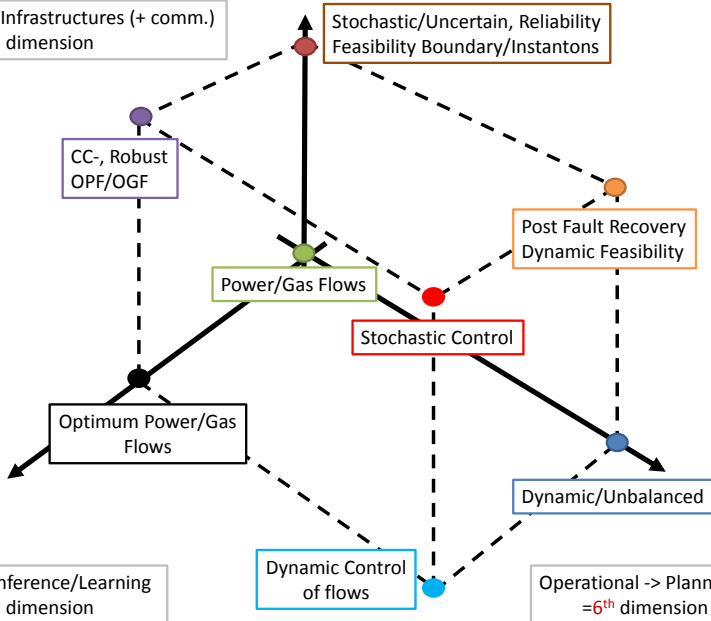
Optimum Power/Gas
Flows

Dynamic/Unbalanced

Data -> Inference/Learning
=5th dimension

Dynamic Control
of flows

Operational -> Planning
=6th dimension



Work in Progress within Grid Science @ LANL – Gas Reliability and Gas-Grid Interdependency

- Steady OGF over graphs with loops
- Other OGF formulations, e.g. max-throughput
- Validating approximations (linearization, adiabatic, etc) vs DNS
- **Stochastic State Estimation** – Learning with Coarse-graining (model reduction) – awareness at the proper level
- **Stochastic (e.g. chance-constrained) OPowerF** aware of gas fluctuations/constraints
- **Stochastic OGasF** aware of **power** (e.g. generation and flow) constraints
- ... other optimization and control formulations, e.g. with line-pack, dynamical phenomena +++ \Rightarrow

└ Gas-Grid Reliability [connecting to "stochastic" Wed]

└ Gas-Grid Coupling, Challenges

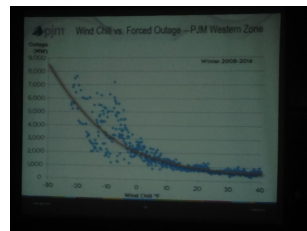
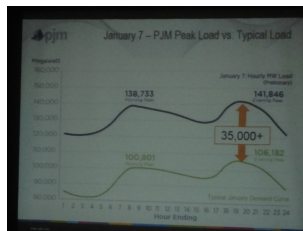
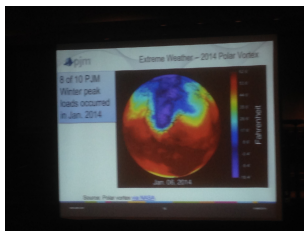
Some concluding thoughts ...

- Gas-Grid Reliability [connecting to "stochastic" Wed]

- Gas-Grid Coupling, Challenges

Climate & Weather

- Terry Boston, PJM CEO — "stolen" slides from HICSS 2015 (last week)

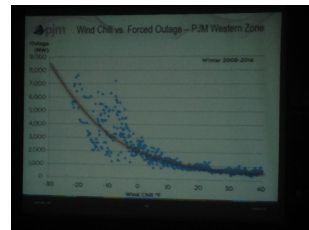
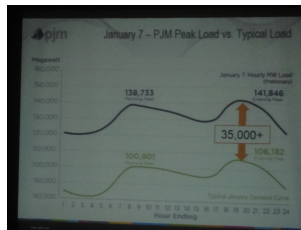
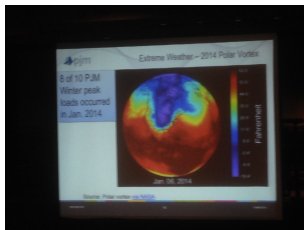


└ Gas-Grid Reliability [connecting to "stochastic" Wed]

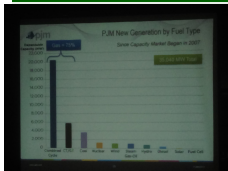
└ Gas-Grid Coupling, Challenges

Climate & Weather

- Terry Boston, PJM CEO — "stolen" slides from HICSS 2015 (last week)



... all of the above combined with ...

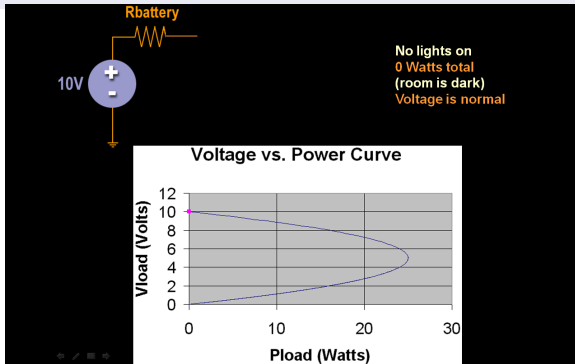


- Availability of Gas is blessing ... but dependence is the nightmare
- ... and renewables "if you wish" [last ... and probably least]
- ... the situation is "dynamic" ... stay tuned ...

Voltage Collapse

- **Voltage Collapse** = Power Flow Eqs. have no solution(s)

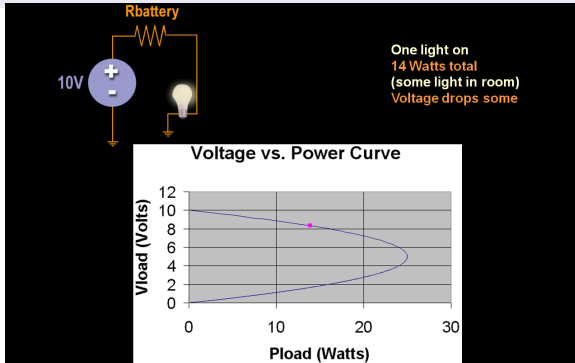
Animation of Voltage Collapse (by P.W. Sauer)



Voltage Collapse

- **Voltage Collapse** = Power Flow Eqs. have no solution(s)

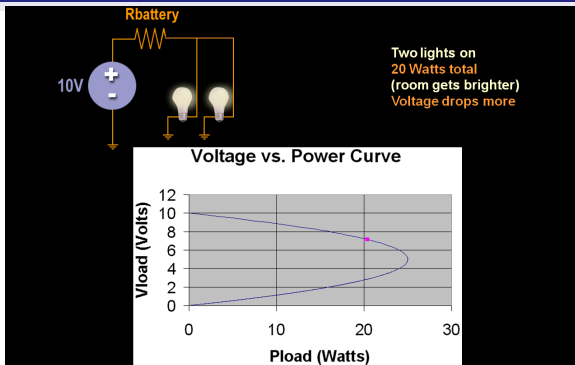
Animation of Voltage Collapse (by P.W. Sauer)



Voltage Collapse

- **Voltage Collapse** = Power Flow Eqs. have no solution(s)

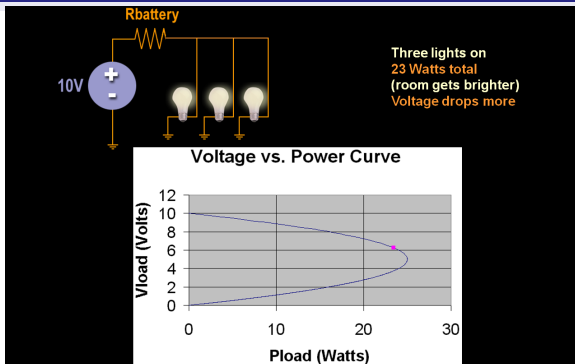
Animation of Voltage Collapse (by P.W. Sauer)



Voltage Collapse

- **Voltage Collapse** = Power Flow Eqs. have no solution(s)

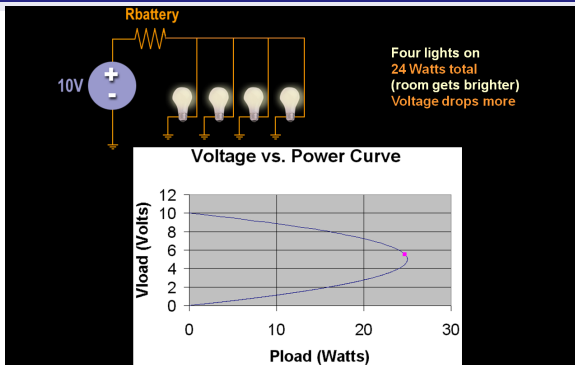
Animation of Voltage Collapse (by P.W. Sauer)



Voltage Collapse

- **Voltage Collapse** = Power Flow Eqs. have no solution(s)

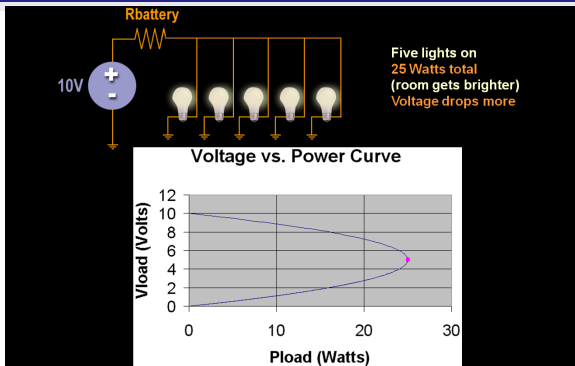
Animation of Voltage Collapse (by P.W. Sauer)



Voltage Collapse

- **Voltage Collapse** = Power Flow Eqs. have no solution(s)

Animation of Voltage Collapse (by P.W. Sauer)



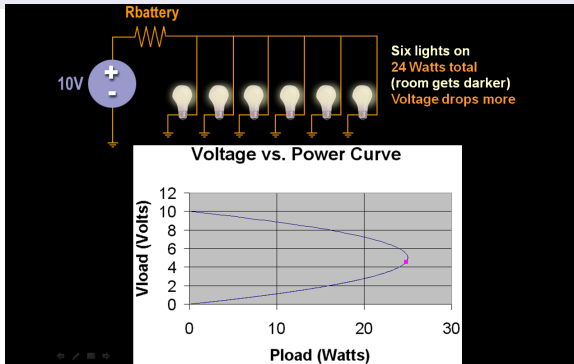
- Fault Induced Delayed Voltage Recovery

- Voltage Collapse

Voltage Collapse

- **Voltage Collapse** = Power Flow Eqs. have no solution(s)

Animation of Voltage Collapse (by P.W. Sauer)

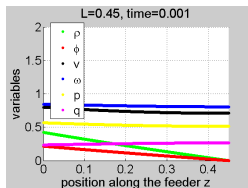


└ Fault Induced Delayed Voltage Recovery

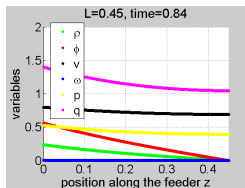
└ Dynamics/Transitions in Distributed Feeder (Aux)

Dynamics/Transitions in Distributed Feeder (III)

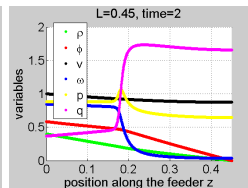
Example of a Short Fault (\downarrow , \uparrow to full recovery) (Movie Recovery)



(a)



(b)



(c)

- (a) Pre fault
- (b) Past voltage drop at the header. Leads to a fully stalled phase.
- (c) Fault is cleared. Front of recovery is advancing towards the tail.

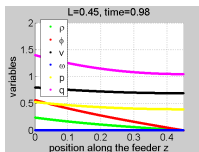
- Fault Induced Delayed Voltage Recovery

- Dynamics/Transitions in Distributed Feeder (Aux)

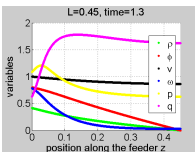
Dynamics/Transitions in Distributed Feeder (IV)

From Stalled to Normal

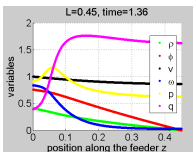
(Movie Recovery)



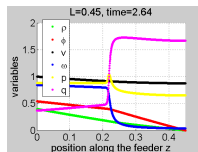
(a)



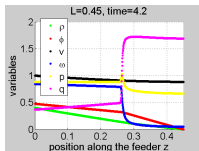
(b)



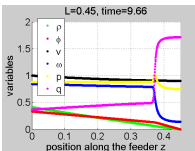
(c)



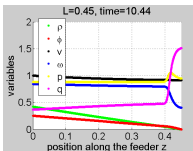
(d)



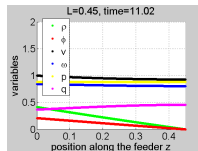
(e)



(f)



(g)



(h)

Of interest: "Soliton"-like shape; voltage profile is (almost) frozen

What can one do at the distribution level to mitigate FIDVR?

- Monitor/learn/model distributed motor parameters
- Control voltage at the head of the line (rise it when needed)
- Distributed reactive control

Why should System Operator worry about FIDVR?

- Simple restoration of the transmission network may not drive the circuits back to a running state.
- A transmission fault \Rightarrow correlated dynamical response in multiple distribution feeders \Rightarrow individual circuits stalled. Specific to each circuit, there is an energy barrier to the transition back to a running state.
- Once a spatially-correlated stalled state exists, the state of the transmission grid has now fundamentally changed.

What can the system operator do about FIDVR and related?

- Consider FIDVR as yet another (and much less analyzed !!) transient stability issue/contingency
- Attempt to predict (monitoring short voltage faults within the transmission) ... and pull it back to normal without relying (or with minimal reliance) on the distribution level protection and response